Preload Modeling, Analysis, and Optimal Design Techniques for Beam/Rod/Cable Element Structures

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This paper presents an optimization method for achieving a predefined structural shape in a space frame. It also presents a general analysis method for structural enforced deformations that are required to preload a structure exactly to a set of prescribed values via turnbuckles or in a structural analysis using MSC/NASTRAN. The analysis and optimization of a cable-tensioned, stabilized, hoop/column-type antenna reflector structure is presented to demonstrate the application of these mehods. The optimization method minimizes the sum of the squares of the displacement or the strain energy in the preloaded structure with respect to a predefined shape. The independent optimization variables used are the enforced deformations.

Nomenclature

A_n = cross-sectional area of nth structural element
$[A_n]$ = $(n_W \times n_W)$ matrices defined in Eqs. (8) and (9); $n=1,2$,
$[B]$ = $(n_C \times n_C)$ influence coefficient matrix that relates $\{w\}$ and $\{d\}$ by Eq. (7)
$\{d\},d_n$ = enforced axial deformation vector and its nth component
$[D]$ = $(n_C \times n_C)$ influence axial deformation coefficient matrix relating $\{d\}$ and $\{e\}$ by Eq. (4)
$[D^*]$ = $[I]+[D]$
$\{e\},e_n$ = axial deformation vector and its nth component
$\{e_A\},\{e_B\}$ = A and B parts of partitioned vectors of $\{e\}$, cf., Eq. (6)
E_n = Young's modulus of nth element
$\{f\}$ = nodal (grid) point force vector
$[F]$ = $(n_W \times n_C)$ influence grid point force matrix
F_n = objective functional
$[G]$ = $(n_C \times n_C)$ influence axial force coefficient matrix relating $\{d\}$ and $\{P\}$ by Eq. (3)
$[G_{AA}],[G_{AB}]$ = partitioned matrices of $[G]$, cf., Eq. (5)
$[H]$ = $(n_C \times n_B)$ matrix defined in Eq. (12)
$[I]$ = identity matrix
$[K]$ = stiffness matrix
ℓ_n = nth element length
N = number of redundant structural elements
n_A = maximum number of axial preloads components that can be arbitrarily prescribed = no. of redundant elements
n_B = number of local and global force (but not moment) equilibrium conditions that the n_c axial force components must satisfy
n_C = total number of axial force/deformation components (= structural elements) included in the analysis, $n_A+n_B=n'_A+n'_B$
n'_A = number of prescribed axial preloads ($\leq n_A$)
n'_B = number of independent axial enforced deformation components, i.e., independent optimization variables, used in the optimization process
n_W = number of grid (nodal) point displacement components used in objective functionals, $=n_{WN}+n_{WT}$
n_{WN} = number of normal nodal displacement components of a reflector surface
n_{WT} = number of tangential nodal displacement components of a reflector surface
$\{P\},P_n$ = axial force matrix and its nth component
$\{P_A\},\{P_B\}$ = $(n_A \times 1)$ and $(n_B \times 1)$ partitioned vectors of $\{P\}$, cf., Eq. (5)
$[R]$ = $(n_B \times n_B)$ – matrix defined in Eq. (14)
u_n = relative axial displacement between the end points of nth structural elements
$\{w\}$ = $(n_W \times 1)$ grid (nodal) displacement vector

I. Introduction

THERE exist several important classes of engineering structures that are internally preloaded prior to application of external loads, such as prestressed concrete structures, cable-tensioned stabilized space antenna structures, etc.

In dealing analytically with this type of structural problem, the first problem one would encounter is how to preload the structure to a given set of self-equilibrated structural element forces via a general-purpose computer code, such as MSC/NASTRAN,[1] for which preloads cannot be directly input into the computer program. The second problem is how to determine, among an infinite number of sets of enforced deformations that give rise to an identical set of self-equilibrated preloads, a proper set of enforced deformations. (The knowledge of enforced deformations is not only necessary in structural analysis of a preloaded structure via MSC/NASTRAN, but is also necessary in actually preloading a hardware structure to a desired set of preloads through structural element length control at the fabrication state and/or use of turnbuckles after the structure is fabricated.) The third problem is how to systematically design pretensions or deformation loads of a complex preinternally loaded structure. In particular, one would be most interested in optimum pretension or enforced deformation design of a high-performance, cable-tensioned stabilized reflector structure such that the reflector surface distortions due to preloads (as well as combined pretension and thermal or dynamic loads) be as small as possible.

The present study is aimed at developing analytical/computational techniques for preload modeling, enforced deformation analysis, and optimum preload/enforced deformation design of beam/rod/cable element structures. In Sec. II, the preload modeling problem is formulated into that of finding a

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set of enforced deformations (DEFORM in NASTRAN input[1]) (or temperatures) that load the structure to the preload values, and two enforced deformation analysis approaches developed. The optimal preload/enforcement design procedure is presented in Sec. III, while an example problem is presented in Sec. IV.

II. Preload Modeling and Enforced Deformation Analysis Procedure

To begin, it is assumed throughout this paper that all preloads (including bending moments) are axial-deformation-induced types.

A. Statically Determinate Structural Analysis Approach

Preload modeling in a general-purpose computer code such as MSC/NASTRAN for which the preloads cannot be directly input into the code is best approached through finding a set of enforced deformations that induce a set of preloads identical to those prescribed. The simplest way to find an equivalent set of enforced deformations (DEFORM) input data in NASTRAN code appears to be using a reduced statically determinate structure and the following analysis steps:

Step 1: Remove redundant structural elements (say, N elements), which are usually cable or rod elements, in order to make the structure statically determinate and apply the element axial forces P_n $(n=1,2,\ldots,N)$, and bending moments and torques, if any, at each pair of conjugate joints where a redundant element was originally attached.

Step 2: Calculate relative axial joint displacements (in the original element axial direction) u_n $(n=1,2,\ldots N)$.

Step 3: Calculate the axial deformations

$$e_n = P_n \ell_n / E_n A_n, \qquad n=1,2,\ldots,N \tag{1}$$

and obtain enforced deformations

$$d_n = u_n - e_n \tag{2}$$

A solution sequence, PLAN 1 (Pre-Load ANalysis sequence 1) for calculation of deformations d_n based on the above analysis approach (approach 1), has been developed by interactively and sequentially using the following computer codes:

1) Develop and use a computer code, say, ANSGEN, for generating the rquired NASTRAN input data as required in the above analysis step 1.

2) Use the NASTRAN code for calculating relative axial displacmeents u_n in step 2.

3) Develop and use the EDEFAN (Enforced DEFormation ANalysis) computer code for performing step 3 calculations.

Alternatively, as will be shown subsequently, one can use influence coefficient methods to find adequate sets of deformations (DEFORM in NASTRAN input) (or temperatures) that load the structure to the prescribed set of preloads. Note that, in general, an infinite number of such enforced deformation sets can be found.

B. Influence Coefficient Approach

Let $[G]$ and $[D]$ be, respectively, the $(n_C \times n_C)$ axial structural element force and relative element end point displacement matrices obtained under a negative unit enforced deformation applied in turn to each of all or appropriately selected individual or groups of structural elements, such as the cable elements in a cable-tensioned stabilized structure. Then,

$$-[G]\{d\} = \{P\} \tag{3}$$

$$-([I]+[D])\{d\} = \{e\} \tag{4}$$

Here, negative signs appear on the left-hand sides of the above equations because a negative enforced deformation component d_n (with all other components of $\{d\}$ zero) induces positive axial force component P_n or deformation component e_n; the nth components e_n and P_n of $\{e\}$ and $\{P\}$ are related to each other by Eq. (1).

In general, matrices $[G]$ and $[D^*] = [I] + [D]$ are singular because not all components of $\{P\}$ and, thus, $\{e\}$ are independent bcause the components of $\{P\}$ or $\{e\}$ are related to one another by local and global equilibrium conditions. Let n_A and n_B be the numbers of independent and dependent components, respectively, and let matrices $[G]$ and $[D^*]$ be partitioned accordingly, so that $n_A + n_B = n_C$ and

$$\begin{bmatrix} G_{AA} & G_{AB} \\ G_{BA} & G_{BB} \end{bmatrix} \begin{Bmatrix} d_A \\ d_B \end{Bmatrix} = -\begin{Bmatrix} P_A \\ P_B \end{Bmatrix} \tag{5}$$

$$\begin{bmatrix} D^*_{AA} & D^*_{AB} \\ D^*_{BA} & D^*_{BB} \end{bmatrix} \begin{Bmatrix} d_A \\ d_B \end{Bmatrix} = -\begin{Bmatrix} e_A \\ e_B \end{Bmatrix}, \quad [D^*] = [I] + [D] \tag{6}$$

For arbitrarily given values of $\{d_B\}$, the A parts of Eq. (5) or (6) can be solved to yield $\{d_A\}$ and thus, there are infinite sets of enforced deformations $\{d\}$ that correspond to given preloads $\{P\}$ that satisfy both local and global equilibrium equations.

III. Optimization Procedures

A. Optimum Set of Enforced Deformations Under a Given Set of Preloads

We shall determine the indefinite ("B-set") components $\{d_B\}$ of enforced deformations in an optimum way such that displacements at some portions of the structure or for the entire structure are minimum. In this optimization process, use of the total or partial strain energy functional or the sum of squares of displacements at some portion of the structure (or for the entire structure) as the objective functional is made.

Let $[B]$ be the displacement coefficient matrix so that

$$\{w\} = -[B]\{d\} \tag{7}$$

where $\{w\}$ is either a total grid (nodal) displacement set in the strain energy functional case, or a partial grid displacement set of "critical" grid points in the case of the square displacement sum functional. Then, the strain energy, V, of the system and square displacement sum S are, respectively,

$$2V = \{w\}^T[K]\{w\} = \{d\}^T[A_1]\{d\}, \quad [A_1] = [B]^T[K][B] \tag{8}$$

$$S = \{w\}^T\{w\} = \{d\}^T[A_2], \qquad [A_2] = [B]^T[B] \tag{9}$$

where superscript T denotes vector or matrix "transpose." Because $[K]\{w\} = [K][B]\{d\} = \{f\} = [F]\{d\}$, $[K][B] = [F]$ where $[F]$ is the grid point force matrix obtained by applying a unit enforced deformation in turn to each of n_C structural elements. Therefore, $[A_1] = [B]^T[F]$.

The independent enforced deformation part, $\{d_A\}$, can be solved from Eq. (5) by solving for $\{d_A\}$ in terms of the dependent part, $\{d_B\}$, to yield

$$\{d_A\} = -[G_{AA}]^{-1}(\{P_A\} + [G_{AB}]\{d_B\}) \tag{10}$$

so that

$$\{d\} = \begin{Bmatrix} d_A \\ d_B \end{Bmatrix} = -\{d'\} + [H]\{d_B\} \tag{11}$$

where

$$\{d'\} = \left\{\begin{array}{c}[G_{AA}]^{-1}\{P_A\}\\ \hdashline \{0\}\end{array}\right\}; \quad [H] = \left[\begin{array}{c}-[G_{AA}]^{-1}[G_{AB}]\\ \hdashline [I]\end{array}\right] \tag{12}$$

Equations similar to Eqs. (10-12) are obtained by using Eq. (6) in place of Eq. (5). Thus, the scalar functionals F_n ($F_1 = 2V$, $F_2 = S$) in Eqs. (8) and (9) can be expressed in terms of $\{d_b\}$ through Eq. (11) as

$$F_n = \{d'\}^T[A_n][d'] - 2\{d_B\}^T[H]^T[A_n]\{d'\} + \{d_B\}^T[H]^T[A_n]^T[H]\{d_B\}, \quad n=1,2 \tag{13}$$

The necessary conditions for F_n to be minimum with respect to $\{d_B\}$ are $\partial F_n/\partial d_{Bi} = 0$ $(i=1,2,\ldots,n_B)$ so that

$$[R]\{d_B\} - [H]^T[A_n]\{d'\} = \{0\}, \quad [R] = [H]^T[A_n][H] \tag{14}$$

Equation (14) can be solved for $\{d_B\}$ to yield

$$\{d_B^m\} = \{d_B\} = [R]^{-1}([H]^T[A_n]\{d'\}) \tag{15}$$

if $[R]$ is nonsingular, i.e., $n_B \le n_W$, where n_W is the total number of nonidentically vanishing grid point displacement components.

The optimal enforced deformation set is then

$$\{d^m\} = \left\{\begin{array}{c} d_A^m \\ \hdashline d_B^m \end{array}\right\}$$

where $\{d_A^m\}$ is obtained by substituting Eq. (15) into Eq. (10).

B. Extension to General Optimum Preload Design Case

In the above, derivations of "optimum" $\{d_B\}$, $\{d_A\}$ and $\{d_B\}$ correspond, respectively, to the independent and dependent parts of the preloads. One can obviously extend this optimization procedure to the general case by specifying a smaller number n_A' $(1 \le n_C - n_W \le n_A' \le n_A)$ of independent components of $\{P_A\}$ and obtain the remaining $n_B'(=n_C - n_A' = n_A + n_B - n_A')$ components through the above optimization techniques. In this case, all equations and formulations given in the preceding sections are applicable provided that n_A and n_B are now replaced by n_A' and n_B' and the "A" and "B" parts of the partitioned vectors and matrices correspond to n_A' and n_B' force or deformation components, and the nonsingular matrix condition now becomes

$$n_B \le n_B' \le n_W \tag{16a}$$

or

$$n_C - n_W \le n_A' \le n_A, \qquad n_C = n_A + n_B = n_A' + n_B' \tag{16b}$$

It should be noted that the "optimum" preload set so obtained may not satisfy all of the constraint conditions

$$P_{Bi} \ge P_{Bi}^* > 0, \qquad i=1,2,\ldots,n_B \tag{17}$$

where P_{Bi}^* are the prescribed minimum tensile load values. Another constraint condition is

$$0 < P_i \le P_i^u, \qquad i=1,2,\ldots,n_C \tag{18}$$

where P_i^u are the ultimate axial forces, are relatively easy to satisfy, and are assumed not to be the control conditions here.

To correct those axial force components that violate constraint conditions (17), we use the redesign rule

$$d_{Bi}^{(k+1)} = d_{Bi}^{(k)} + \sum_{j=1}^{n_B} \tilde{G}_{BBij}^{-1}\left(P_{Bj}^{(k+1)} - P_{Bj}^{(k)}\right)$$

$$P_{Bj}^{(k+1)} = P_{Bj}^{(k)} \quad \text{if} \quad P_{Bj}^{(k)} \ge P_{Bj}^*$$

$$P_{Bj}^{(k+1)}Bj = P_{Bj}^* \quad \text{if} \quad P_{Bj}^{(k+1)}Bj \le P_{Bj}^* \tag{19}$$

where superscripts k stand for the kth iterative solution and $\tilde{G}_{BBij}$ is the i, jth element of the matrix

$$[\tilde{G}_{BB}] = [G_{BB}] - [G_{AB}]^T[G_{AA}]^{-1}[G_{AB}] \tag{20}$$

As is usually the case in an optimality criteria-based optimum design,[2] there is no assurance that an "optimum" solution obtained from the preceding optimization procedure is truly optimum. Nevertheless, such an "optimum" solution is useful because if it is not truly optimum, it represents a nearly optimum one.

C. Computer Code System (ANSTAN) and Solution Sequences (PLAN 1 and 2)

The ANSTAN code system consists of MSC/NASTRAN code and the newly developed FORTRAN codes—ANSGEN, EDEFAN, and DATPRO.

Corresponding to two preload/enforced deformation modeling/analysis procedures developed, two automated solution sequences, PLAN 1 and PLAN 2, have been developed and implemented in the VAX 11/780 model computer system at MRJ, Inc. This is done by using ANSGEN code to create a pertinent NASTRAN input data file and NASTRAN code to calculate the relative displacements or influence coefficients, and the EDEFAN computer code to calculate the enfored deformations or perform an optimum preload design. The PLAN 1 solution sequence is applicable to the case in which the cable pretension loads are already known while PLAN 2, in addition to this, can also be used in preload design. The optimum preload/enforced deformation design procedures presented in Sec. III are also implemented in EDEFAN code.

The foregoing solution sequences, PLAN 1 an PLAN 2, are further extended to form two complete solution sequences for preload analysis/design and nonlinear elastic or thermoelastic structural analyses and plottings of preloaded beam/rod/cable element structures in general and multiple-sectored, offset vertex parabolic reflector subsystem structures in particular. These are done by having the computer code system automatically assemble the required data generated from the ANSGEN and EDEFAN codes (for enforced deformations) for the subsequent NASTRAN nonlinear thermoelastic analysis (via SOL 64) of preloaded structures. These results are then post-processed and the required NASTRAN data for plotting the final deformed structural configuration that is overlaid with the preloaded structural configuration are automatically assembled by using a newly developed Fortran postprocesser, DATPRO. Therefore, using this NASTRAN-interfaced, automated preload, and thermoelastic structural analysis/design capability of antenna structures, ANSTAN, one is required to punch the appropriate command file names only twice for both preload design/analysis and nonlinear preloaded thermoelastic structural analysis. If, in addition, the final overlaid structure plots are to be obtained, three command file names are required to be punched in using the VAX on-line operations mode.

IV. Example

Consider a reflector structure as shown in Fig. 1. Due to cyclic symmetry, only one bay from a total of 24 bay structural subassemblies is shown here. All structural elements except for the outer hoop elements, centrally located column elements, and the horizontal elements adjacent to the column elements, which are beam-column elements, are cable elements. The joints where the upper suspension elements, surface elements, and hoop elements are joined together all lie on the reflector parabolic surface. This type of reflector structure in which only hoop and centrally located column elements can carry bending moments and compressive forces is called the hoop/column type in Ref. 3.

Unless otherwise noted, the objective functional used throughout this example problem is the sum of the squares of reflector surface grid point displacment components.

A. Enforced Deformation Analysis Under Prescribed Cable Pretension Loads

The maximum number of cable element forces one is allowed to prescribe is $n_A = 10$. Let these forces be chosen to be those of all five upper cable elements (Nos. 6-10 in Fig. 1) and five reflector surface cable elements (Nos. 1-5 in Fig. 1). With this, the forces ($n_B = 5$) in the remaining five cable elements (elements Nos. 11-15) are uniquely determined by the local and global equilibrium conditions. Such a set of self-equilibrated cable pretension loads is obtained, and is shown in Table 1.

As discussed previously, there are infinite sets of enforced deformations that can load up the structure to a set of prescribed pretension values by arbitrarily assigned enforced deformation values in the $n_B = 5$ cable elements (Nos. 11-15) and determine the values of the remaining $n_A = 10$ deformation components (Nos. 1-10) and five cable preloads (Nos. 11-15) according to the procedure described in Sec. II.B. Tables 1 and 2 show both the assigned values and the results of enforced deformations and the corresponding normal and tangential displacements under the ten prescribed pretension load components and five prescribed enforced deformation components whose values are set equal to the corresponding axial deformation values [i.e., $d_n = e_n = P_n \ell_n / E_n A_n$; $n = 11,...,15$; cf., Eq. (1)]. Large induced normal displacements are seen, and it is desirable to obtain a set of enforcd deformations that minimize the reflector surface deflections (particularly the normal displacement components).

Optimum Enforced Deformation Analysis—I: ($n_B = n_W = 5$ case)

Using the optimization techniques developed in Sec. III.A and the PLAN 2 solution sequence of the ANSTAN computer code system, the optimum enforced deformation (distribution) set was obtained and is presented in Table 2 for the same pretension loads given in Table 1. The objective functional used in this optimization was the sum of the squares of the five normal displacement components of the five movable reflec-

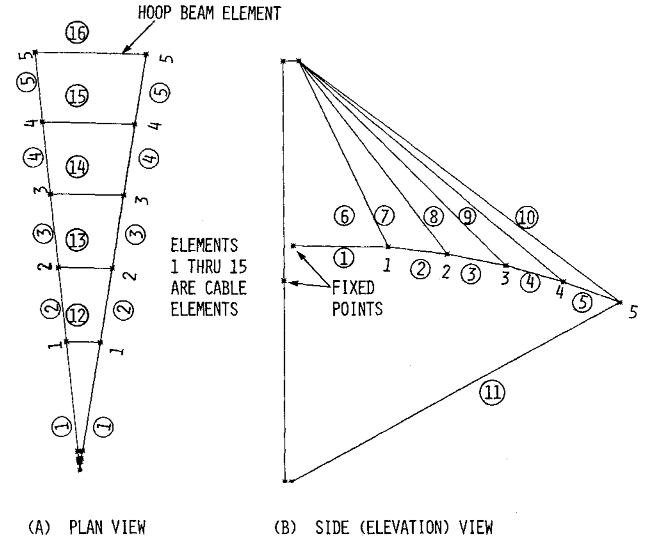

Fig. 1 One-bay skeletal structural assembly of hoop/column-type reflector subsystem.

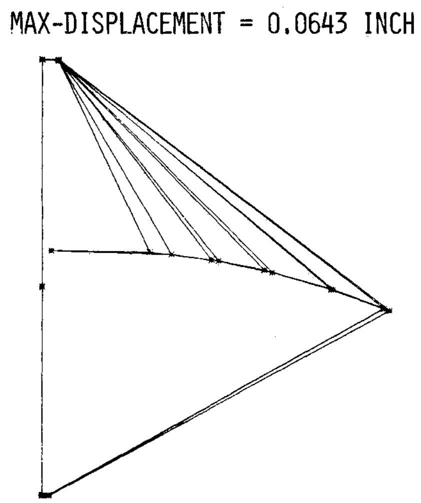

Fig. 2 Preloaded structural configuration overlaid with undeformed configuration.

Table 1 Enforced deformation analysis results—I ($n_A = 10$, $n_B = 5$, $n_W = 0$)

Preloads					
0.96590E+01	0.14962E+02	0.22959E+02	0.35063E+02	0.51807E+02	0.18377E+01
0.34873E+01	0.70770E+01	0.12541E+02	0.71816E+02	0.12802E+03	0.16910E+02
0.21221E+02	0.23878E+02	0.20668E+02			
Enforced deformations					
−0.14171E+00	0.26128E+00	0.59133E+00	0.47100E+00	−0.17570E+00	−0.66434E+01
−0.41053E+01	0.15408E+01	−0.29713E+00	−0.61879E+00	−0.51139E+00	−0.24230E−02
−0.48002E−02	−0.73590E−02	−0.80310E−02			
Normal displacements					
−0.72796E+01	−0.50340E+01	−0.20325E+01	−0.28429E+00	−0.21293E+00	
Tangential displacements					
−0.60277E+00	−0.83083E+00	−0.50045E+00	−0.92649E−01	−0.25235E+00	

NOTE: Those underlined are prescribed values. Forces and displacements are in pound and inch units, respectively.

tor surface grid points (Nos. 1-5 in Fig. 1). A similar optimum set of enforced deformations under a different prescribed set of admissible pretension loads shown in Table 3 were also obtained and the analytical results presented in Table 3. Because there are five optimality equations ($n_W = 5$) and five independent variables (enforced deformations), one is able to achieve the zero normal displacement design for both cases of preloads.

As a check case, the enforced deformation values (DEFORM input data in MSC/NASTRAN) listed in Table 3 were used in a NASTRAN check run. Virtually the same analytical results as those shown in Table 3 were obtained. A corresponding NASTRAN plot of the preloaded structural configuration that is overlaid with the undeformed configuration is shown in Fig. 2.

In general, the number (n_{WN}) of normal displacement components of reflector surface grids may be larger than that (n_B) of the dependent cable force components, or one may wish to also include some or all tangential displacement components (N_{WT}) of the reflector surface grid points in the optimization so that $n_W = n_{WN} + n_{WT} > n_B$. Such an optimum design example case is presented below mainly for demonstration purposes because the enforced deformation sets obtained in Tables 2 and 3 are already exceedingly satisfactory from a practical reflector design viewpoint.

Optimum Enforced Deformation Analysis—III: (case for $n_B = 5$ and $n_W = 10$)

Let us consider the case in which the objective functional is the square sum of all five normal and five tangential displacement components of the movable reflector surface grid points so that $n_w = 10$. This objective functional was minimized with respect to the five ($n_B = 5$) enforced deformation components corresponding to cable elements 11-15 in Fig. 1. The analytical optimum results under the same preload set as that given in Tables 1 and 2 are presented in Table 4. Although none of the displacments shown is zero, they are all small, certainly much smaller than those of the unoptimized case shown in Table 1.

B. Optimum Preload/Enforced Deformation Design

To further optimize the preload/enforced deformation design, we shall use the optimization technique presented in Sec. III.B. The allowable number (n_B) of independent optimization variables (enforced deformations) must be subjected to the restriction, arising from the nonsingular matrix condition, (16a) or (16b), such that $5 \leq n_B' \leq n_W \leq 10$ ($n_W = n_{WN} + n_{WT}$). The following two subcases are considered first:

Subcase 1: $n_B' = 9$ ($n_A' = 6$), $n_W = 9$ ($n_{WN} = 5$, $n_{WT} = 4$) i.e., all normal and tangential displacement components except for the fifth tangential component at the outer tip of reflector surface are included in the optimization; the six specified pretension values are those of cable elements 5-10.

Subcase 2: $n_B' = n_W = 10$ ($n_A' = 5$), i.e., all ten displacement components are included, and the five prescribed cable element forces belong to cable elements 6-10.

The optimum analytical results corresponding to these two subcases are given in Tables 5 and 6, respectively. In the former case, all cable preloads are seen to be tensile ones and the nine displacement components included in the objective

Table 2 Optimum enforced deformation analysis results—I
($n_A = 10$, $n_B = 5$, $n_{WN} = 5$)

		Preloads			
0.96590E+01	0.14962E+02	0.22959E+02	0.35063E+02	0.51807E+02	0.18377E+01
0.34873E+01	0.70770E+01	0.12541E+02	0.71816E+02	0.12802E+03	0.16909E+02
0.21221E+02	0.23878E+02	0.20668E+02			
		Enforced deformations			
0.24579E+00	−0.21181E+00	−0.25235E+00	0.68663E−01	−0.10651E+00	0.10352E+00
0.70195E−02	−0.21363E+00	−0.18979E+00	−0.47948E+00	−0.62140E+00	0.64177E−01
0.90053E−02	−0.54055E−01	−0.31120E−01			
		Normal displacements			
−0.36206E−13	−0.34303E−13	−0.15508E−14	−0.53659E−14	0.13730E−13	
		Tangential displacements			
0.25599E+00	0.53605E−01	−0.18422E−00	−0.93023E−01	−0.16637E+00	

NOTE: Those underlined are prescribed values.

Table 3 Optimum enforced deformation analysis results—I
($n_A = 10$, $n_B = 5$, $n_w = n_{WN} = 5$)

		Preloads			
0.81132E+01	0.22893E+02	0.40020E+02	0.63867E+02	0.95923E+02	0.29448E+01
0.65699E+01	0.13402E+02	0.23668E+02	0.38816E+02	0.11780E+03	0.51189E+02
0.47978E+02	0.48863E+02	0.40841E+02			
		Enforced deformations			
0.63579E+00	−0.44237E+00	−0.47590E+00	0.23731E+00	−0.27748E+00	0.29295E+00
0.11051E+00	−0.29408E+00	−0.14323E+00	−0.34143E+00	−0.58321E+00	0.16054E+00
0.45087E−01	−0.74223E−01	−0.46981E−02			
		Normal displacements			
0.25470E−13	−0.31745E−14	0.94412E−14	−0.13560E−13	0.24359E−13	
		Tangential displacements			
0.64526E+00	0.21720E+00	0.23341E+00	−0.45013E−01	−0.17113E+00	

NOTE: Those underlined are prescribed values.

Table 4 Optimum enforced deformation analysis results—III
($n_A = 10$, $n_B = 5$, $n_{WN} + n_{WR} = 10$)

		Preloads			
0.96590E+01	0.14962E+02	0.22959E+02	0.35063E+02	0.51807E+02	0.18377E+01
0.34873E+01	0.70770E+01	0.12541E+02	0.71816E+02	0.12802E+03	0.16909E+02
0.21221E+02	0.23878E+02	0.20668E+02			
		Enforced deformations			
0.17449E+00	−0.15577E+00	−0.18747E+00	0.49832E−01	−0.81108E+01	0.67860E+01
−0.27571E−01	−0.17967E+00	−0.13966E+00	−0.37869E+00	−0.70098E+00	0.45591E−01
0.60890E−02	−0.41569E−01	−0.28688E−01			
		Normal displacements			
−0.24919E−03	−0.32614E−01	−0.11114E−01	0.60344E−01	0.15406E+00	
		Tangential displacements			
0.18453E+00	0.36900E−01	−0.13770E+00	−0.63560E−01	−0.10417E+00	

NOTE: Those underlined are prescribed values.

Table 5 Optimum enforced deformation analysis results—I
($n'_A = 6$, $n'_B = 9$, $n_{WN} = 5$, $n_W = 9$, $n_{WT} = 4$)

		Preloads			
0.96589E+01	0.14962E+02	0.22959E+02	0.35063E+02	0.51807E+02	0.18377E+01
0.34873E+01	0.70770E+01	0.12541E+02	0.71816E+02	0.12802E+03	0.16910E+02
0.21221E+02	0.23878E+02	0.20668E+02			
		Enforced deformations			
−0.96676E−02	−0.95858E−02	−0.14710E−01	−0.22466E−01	−0.19947E+00	−0.23454E−01
−0.31796E−01	−0.56288E−01	−0.10369E+00	−0.47948E+00	−0.62140E+00	−0.24225E−02
−0.48016E−02	−0.73590E−02	−0.80314E−02			
		Normal displacements			
0.15217E−13	0.14957E−13	0.14663E−13	0.14249E−13	0.60715E−17	
		Tangential displacements			
−0.20547E−16	−0.26304E−14	0.37463E−14	0.46570E−14	−0.16637E+00	

NOTE: Those underlined are prescribed values. The last tangential displacement component was not included in the optimum design.

Table 6 Optimum preload/enforced deformation analysis results—II
($n'_A = 5$, $n'_B = n_W = 10$)

		Preloads			
−0.26286E+04	−0.37280E+03	−0.21380E+03	−0.13833E+03	−0.87330E+02	0.18377E+01
0.34873E+01	0.70770E+01	0.12541E+02	0.71816E+02	0.28475E+02	0.86472E+04
0.60697+03E	0.27365E+03	0.15855E+03			
		Enforced deformations			
0.26309E+01	0.23884E+00	0.13698E+00	0.88631E−01	0.55959E−01	−0.23454E−01
−0.31796E−01	−0.56288E−01	−0.10369E+00	−0.31905E+00	−0.11473E+00	−0.12390E+01
−0.13734E+00	−0.84334E−01	−0.61609E−01			
		Normal displacements			
0.87195E−12	−0.14085E−12	−0.10778E−13	0.15005E−13	0.34173E−13	
		Tangential displacements			
0.18457E−13	0.24058E−13	0.23484E−13	0.14936E−13	0.29106E−13	

NOTE: Those underlined are prescribed values.

functional are zero and the remaining one is less than 0.2 in. As for the minimum pretension value condition, Eq. (17), it is a trivial one to satisfy because one can scale up or down all quantities in Table 5 (or any other tables except for Table 6 presented in this example problem) by a common factor to satisfy this condition.

In the latter (Table 6) case, all reflector grid point displacements are seen to be practically zero, but the surface cable element forces are all compressive. This is not totally unexpected because physically, it is impossible to maintain zero normal and tangential displacements at the outer tip grid point if all reflector surface cable elements are in tension, unless outer hoop (beam) elements are also under an appropriate enforced deformation loading.

The optimum design case study in which the outer hoop (beam) element is included in the optimization was also made. Some analytical results are presented in Table 7. The last (sixteenth) preload and enforced deformation components correspond to the outer hoop beam element. It is seen that all ten calculated cable preloads as well as the five prescribed are in

Table 7 Optimum enforced deformation analysis results—I ($n'_A = 6$, $n'_B = n_W = 10$)

		Preloads			
0.96587E+01	0.14962E+02	0.22959E+02	0.35063E+02	0.51807E+02	0.18377E+01
0.34873E+01	0.70770E+01	0.12541E+02	0.71816E+02	0.12802E+03	0.16910E+02
0.21221E+02	0.23878E+02	0.20668E+02	−0.83563E+03		
		Enforced deformations			
−0.96674E−02	−0.95857E−02	−0.14710E−01	−0.22466E−01	−0.33197E−01	−0.23454E−01
−0.31796E−01	−0.56288E−01	−0.10369E+00	−0.31905E+00	−0.51582E+00	−0.24226E−02
−0.48016E−02	−0.73590E−02	−0.80314E−02	0.40273E−01		
		Normal displacements			
0.18793E−13	0.18657E−13	0.18435E−13	0.18045E−13	−0.12486E−15	
		Tangential displacements			
−0.24625E−14	0.26004E−14	0.47436E−14	0.58334E−14	−0.23749E−15	

NOTE: Those underlined are prescribed values.

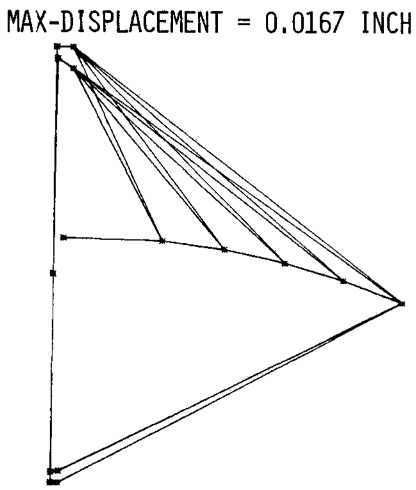

Fig. 3 Undeformed and optimally preloaded (deformed) structural configurations (exaggerated displacement plot).

tension, while the hoop beam axial force is in compression. Also, all reflector surface grid point displacement components are seen to be zero. Thus, we have achieved a zero reflector surface displacement, preload/enforced deformation design. The enforced deformation values given in Table 7 were then used in a NASTRAN computer run to load up the structure and the response and preload results compared with those of the EDEFAN computer run. Both computer run results agree completely. An exaggerated NASTRAN structure plot is shown in Fig. 3.

The computer CPU time in the EDEFAN computer run for the last example (Table 7) case of optimum preload/enforced deformation calculations was minimal (only 7.82 s on the VAX 11/780 computer system). The NASTRAN run that generated influence coefficients required for the preceding calculations via EDEFAN consumed 54.54 s of CPU time on the same computer system. Thus, the preload/enforced deformation analysis/optimum design techniques and computational procedures developed herein are seen to be very efficient.

V. Summary and Conclusion

An efficient, enforced deformation-based optimization technique has been developed for minimizing the distortions with respect to a predefined geometric shape of a preloaded space frame structure. Also devloped were an enforced deformation analysis method of a preloaded structure and an associated structural analysis/optimization computer code system, ANSTAN, consisting of a newly developed preprocessor, ANSGEN, MSC/NASTRAN (used to generate the required influence coefficient matrices), and newly developed enforced deformation analysis/optimization code, EDEFAN, and postprocessor, DATPRO, for nonlinear thermoelastic analysis of preloaded structures and deformed structural configuration plots via NASTRAN. As demonstrated in an example problem of a simple hoop/column-type, cable-tensioned, stabilized reflector structure, the developed analytical technqiues and computational tools are capable of achieving or obtaining the following:

1) Achieving a zero (or nearly zero) structural (reflector surface) distortion design with respect to a predefined structural geometric shape.

2) Obtaining an enforced deformation set of a preloaded structure that not only minimizes the structural distortions, but also can be used to actually preload the structure exactly to the prescribed values a) via structural element length control in the structural fabrication stage or length adjustment using turnbuckles in the postfabrication stage, and b) in a structural analysis using MSC/NASTRAN.

3) Achieving the preceding optimization or analysis in a highly efficient manner based on minimal computer CPU time (8 s) for an optimization case run of the example problem using MRJ's VAX 11/780 model computer system.

Such enforced deformation-based optimization is possible because of the fact that an infinite number of sets of enforced deformations can be found to load the structure to a prescribed self-equilibriated set of preloads due to the existence of arbitrarily prescribable n_B components (designated as "B-set" components) of enforced deformations, where n_B is equal to the number of local and global equilibrium conditions on forces (but not moments). Based on this, a systematic enforced deformation design procedure has been formulated that optimizing certain objective functionals with respect to the "B-set" of enforced deformation components. Two objective functionals were considered. The first one is the sum of the squares of displacement components at some critical locations (such as surface locations in a reflector structure), and the second is the partial of total strain energy of the structure. The objective of this type of optimum design is to minimize the distortions (deflections) of critical portions of the structure or system under preloading (or subsequent mechanical and/or thermal loading). This type of optimization formulation is useful in applications in which reflector surface distortions are to be kept as small as possible.

The foregoing optimum design procedure has been further extended to the general case of optimum preload/enforced deformation design in which only a small number of axial preloads are presecribed. The remaining n_B' ($\geq n_B$) optimum preload components (which must be tensile for cable elements) are obtained from equilibrium equations and by minimizing one of the two aforementioned objective functionals with respect to larger numbers (n_B') of enforced deformation components compared with the n_B components used in the foregoing optimization procedure.

A preload analysis/design example problem of a simple hoop/column-type reflector subsystem structure was presented for demonstration purposes. Through use of the optimum preload/enforced deformation design procedures formulated herein, one was able to achieve a zero reflector surface displacement design of the structure under an optimum set of preloads axial enforced deformations. Based on this result, it appears that such a zero (or nearly zero) reflector surface distortion design can be achieved for a more complex cable-tensioned, stabilized reflector structure such as those hoop/column reflector designs presented in Ref. 3 or other types of preloaded reflector structures.

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